Discrete Math Relations Test Name ______________________________

1. What is a binary relation?
   a. A set of ordered pairs
   b. A set of unordered pairs
   c. A function that maps elements from one set to another
   d. A graph with exactly two vertices

2. Which of the following is *not* a property that a relation can have?
   a. Reflexive
   b. Symmetric
   c. Transitive
   d. Commutative

3. A relation R on a set A is an equivalence relation if and only if it is:
   a. Reflexive and symmetric
   b. Symmetric and transitive
   c. Reflexive and transitive
   d. Reflexive, symmetric and transitive

4. Given a binary relation R, the inverse relation R^{-1} is defined as:
   a. $\{(a, b) : (b, a) \in R\}$
   b. $\{(a, b) : (a, b) \notin R\}$
   c. $\{(b, a) : (a, b) \notin R\}$
   d. $\{(a, a) : a \in R\}$

5. If R is a relation from set A to B and S is a relation from B to C, the composition $S \circ R$ is:
   a. $\{(a, c) \in A \times C : (a, b) \in R \text{ and } (b, c) \in S \text{ for some } b \in B\}$
   b. $\{(a, b) \in A \times B : (a, c) \in R \text{ and } (c, b) \in S \text{ for some } c \in C\}$
   c. $\{(a, c) \in A \times C : (b, a) \in R \text{ and } (c, b) \in S \text{ for some } b \in B\}$
   d. $\{(b, c) \in B \times C : (a, b) \in R \text{ and } (a, c) \in S \text{ for some } a \in A\}$

6. Define a relation R on the set of integers {1, 2, 3, ..., 10} by:

$$a \; R \; b \text{ if and only if } a \equiv b \pmod{4}$$

Write this relation as a set of ordered pairs. Is R an equivalence relation? Justify your answer. If R is an equivalence relation, list all its equivalence classes.

Solutions

Question 1

a. A set of ordered pairs

Explanation:

A **binary relation** between two sets AA and BB is a collection of ordered pairs (a,b)(a,b) where $a \in Aa \in A$ and $b \in Bb \in B$. In other words, it is a subset of the Cartesian product $A \times BA \times B$.

- **Option b** is incorrect because a binary relation specifically involves *ordered* pairs, not unordered pairs.
- **Option c** is incorrect because while a function is a special type of binary relation (one where each element of AA is related to exactly one element of BB), not all binary relations are functions.
- **Option d** is incorrect because a binary relation is not inherently a graph, though it can be *represented* as a directed graph in some cases.

Thus, the most general and correct definition is that a binary relation is a **set of ordered pairs**.

Question 2

d. Commutative

Explanation:

A relation on a set can have several standard properties, including:

- **Reflexive** (a relation R where every element is related to itself: $\forall$ a, (a, a) $\in$ R.
- **Symmetric** (a relation R where if (a, b) $\in$ R, then (b, a) $\in$ R.
- **Transitive** (a relation R where if (a, b) $\in$ R and (b, c) $\in$ R, then (a, c) $\in$ R.

However, commutative is not a standard property of relations. Instead, commutativity is a property of binary operations (e.g., addition and multiplication, where $a + b = b + a$).

Thus, the correct choice is d. Commutative, as it does not describe a property of relations.

Question 3

d. Reflexive, symmetric, and transitive

Explanation:

A relation R on a set A is called an equivalence relation if and only if it satisfies all three of the following properties:

1. **Reflexive**: Every element is related to itself ($\forall a \in A, (a, a) \in R$).
2. **Symmetric**: If $(a, b) \in R$, then $(b, a) \in R(b,a)$.
3. **Transitive**: If $(a, b) \in R$ and $(b ,c) \in R$, then $(a, c) \in R$.

All three conditions must hold for a relation to be an equivalence relation. Missing any one of them means the relation is not an equivalence relation.

Why not the other options?

- a. Reflexive and symmetric → Missing transitivity (e.g., "is a friend of" may not be transitive).
- b. Symmetric and transitive → Missing reflexivity (e.g., the empty relation is symmetric and transitive but not reflexive).
- c. Reflexive and transitive → Missing symmetry (e.g., "≤" on integers is reflexive and transitive but not symmetric).

Thus, the correct choice is d. Reflexive, symmetric, and transitive.

Question 4

a. $\{(a, b) : (b, a) \in R\}$

Explanation:

The inverse relation R^{-1} of a binary relation R is obtained by reversing all the ordered pairs in R.

- Formally, if R is a relation from set A to set B, then:

  $$R^{-1} = \{(b, a) \mid (a, b) \in R\}$$

- In the given options, (a) matches this definition, just written as $\{(a, b) \mid (b, a) \in R\}$ (which is equivalent since (a, b) in R^{-1} corresponds to (b, a) in R).

Why not the other options?

- (b) Incorrect: This describes the complement of R, not the inverse.
- (c) Incorrect: This is a mix of inverse and complement, which is not a standard operation.
- (d) Incorrect: This describes the identity relation, not the inverse.

Thus, the correct choice is a.

Question 5

a. {(a, c) ∈ A × C : (a, b) ∈ R and (b, c) ∈ S for some b ∈ B}

Explanation:

The **composition** $S \circ R$ of two relations R (from A to B) and S (from B to C) is defined as:

$$S \circ R = \{(a, c) \mid \exists\, b \in B \text{ such that } (a, b) \in R \text{ and } (b, c) \in S\}$$

- This means we "chain" the relations by finding intermediate elements b ∈ B that connect a ∈ A to c ∈ C.

Why not the other options?

- **(b)** Incorrect: The condition is wrongly stated (mixes b and c, and the output is in A × B instead of A × C).
- **(c)** Incorrect: Reverses the order of pairs ((b, a) instead of (a, b)), which is not how composition works.
- **(d)** Incorrect: Describes a subset of B × C, not A × C, and the condition is misaligned.

Thus, the correct choice is **a**.

Question 6

1. Defining the Relation R as a Set of Ordered Pairs

The relation R is defined on the set A = {1, 2, 3, …,10} such that:

$$a \, R \, b \iff a \equiv b \pmod 4$$

This means a and b are related if they leave the same remainder when divided by 4.

The possible remainders modulo 4 are 0, 1, 2, 3. Thus, we group the numbers in A based on their remainder:

- Remainder 0: 4, 8
- Remainder 1: 1, 5, 9
- Remainder 2: 2, 6, 10
- Remainder 3: 3, 7

The relation R consists of all ordered pairs (a,b) where $a \equiv b \pmod 4$. Therefore:

R= {(1, 1), (1, 5) , (1, 9), (5,1), (5, 5), (5, 9), (9, 1), (9, 5), (9, 9),
(2, 2), (2, 6), (2, 10), (6, 2), (6, 6), (6, 10), (10 ,2), (10, 6), (10, 10),
(3, 3), (3, 7), (7, 3), (7, 7),
(4, 4), (4, 8), (8, 4), (8, 8)}

2. Checking if R is an Equivalence Relation

An equivalence relation must satisfy three properties: reflexivity, symmetry, and transitivity.

- **Reflexive**: Yes, because a ≡ a(mod4) for all a ∈A. (Every element is related to itself.)
- **Symmetric**: Yes, because if a ≡ b(mod4), then b ≡ a(mod4). (If (a, b) ∈ R, then (b, a) ∈ R.)
- **Transitive**: Yes, because if a ≡ b(mod4) and b ≡ c(mod4), then a ≡ c(mod4). (If (a, b) ∈ R and (b, c) ∈ R, then (a, c) ∈ R.)

Since R satisfies all three properties, it is an equivalence relation.

3. Listing the Equivalence Classes

The equivalence classes are the sets of numbers that share the same remainder modulo 4:

- [1] = {1, 5, 9} (numbers ≡ 1 mod 4)
- [2] = {2, 6, 10} (numbers ≡ 2 mod 4)
- [3] = {3, 7} (numbers ≡ 3 mod 4)
- [4] = {4, 8} (numbers ≡ 0 mod 4)

Final Answer:

1. The relation R is:

   R= {(1, 1), (1, 5) , (1, 9), (5,1), (5, 5), (5, 9), (9, 1), (9, 5), (9, 9), (2, 2), (2, 6), (2, 10), (6, 2), (6, 6), (6, 10), (10 ,2), (10, 6), (10, 10), (3, 3), (3, 7), (7, 3), (7, 7), (4, 4), (4, 8), (8, 4), (8, 8)}

2. Yes, R is an equivalence relation because it is reflexive, symmetric, and transitive.

3. The equivalence classes are:

   [1] = {1, 5, 9}, [2] = {2, 6, 10}, [3] = {3, 7}, [4] = {4, 8}