Discrete Math Counting Test Name ____Answer Key_(22 points)_____

1. Let $S = \{1, 4, 9, 16, 25, 36, 49\}$. How many subsets does S have in total? How many subsets contain {4, 16, 36}? How many subsets of S of cardinality 4 contain at least one odd number?

   With 7 elements, there are $2^7 = 128$ subsets of S. The number of subsets that contain {4, 16, 36} is equal to $|P(\{1, 9, 25, 49\})| = 2^4 = 16$. The number of subsets of S with cardinality 4 is 7 choose 4 = $\frac{7!}{3!4!} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2} = 35$. Since there are only 3 even numbers in S, all of these 35 subsets contain at least one odd number. **6 points**

2. Students at ACC can participate in student clubs during their Archer's period. There are 5 clubs (including the Programming Club) that meet on Mondays and Fridays, and 9 clubs that meet on Tuesdays, Wednesdays, and Thursdays.

   a. If students have to participate in all meetings of their chosen club to be members, how many different combinations of clubs does a student have to choose from?

      Students have 5 choices for a Monday/Friday club and 9 choices for a Tuesday/Wednesday/Thursday club, for a total of 5 times 9 or 45 combinations of club choices. **2 points**

   b. What if students only need to attend a club once a week to be a member. How many options do they have?

      Students under this scenario can choose to either be members of one or two different clubs on Mondays and Fridays, giving them 5 choose 1 + 5 choose 2 = 15 options, and they can choose either one, two, or three different clubs Tuesdays, Wednesdays, and Thursdays, giving them 9 choose 1 plus 9 choose 2 plus 9 choose 3 = 129 options. Using the multiplicative principle for these independent events give students a total of 15 · 129 = 1935 different club membership choices. **2 points**

3. In an attempt to clean up your room, you have purchased a new wall mounting bookshelf to put some of the 15 books by different authors that you have stacked in a corner. The new bookshelf is large enough to hold 8 books.

    a. How many ways can you choose 8 of your 15 books for your bookshelf and then arrange them alphabetically by author on your shelf?

    There are 15 choose 8 = 6435 different ways to choose 8 of your 15 books, and exactly one way (alphabetically by author) to arrange each of these. **2 points**

    b. If you arrange the 8 books randomly, instead of my author, how many arrangements are there for a given set of 8 books? How many possible arrangements altogether are there for 8 of 15 randomly selected books randomly arranged?

    For each of your 6435 possible sets of 8 books, there are 8! = 40320 total possible arrangements, giving a total of 6435 times 40320, or 259,459,200 possible ways to randomly arrange a random set of 8 of your books on the new shelf. **4 points**

4. Bruni's Pizza in Hammonton, New Jersey makes the best pizza in the world. They also offer 16 different choices for toppings.

    a. How many 2-topping pizzas could you order at Bruni's?

    There are 16 choose 2, or 120 possible 2-topping pizzas at Bruni's. **2 points**

    b. How many different pizza experiences are possible altogether, if you can order plain cheese, or one, two, or three toppings?

    Plain cheese is a single choice, plus 16 single topping choices, plus the 120 2-topping choices, plus 16 choose 3, or 560 3-topping pizzas, for a total of 1 + 16 + 120 + 560 = 697 total pizza experiences. **2 points**

    c. Bruni's wants NOVA Web Development to build them a new website listing their 16 toppings choices in 4 equally sized columns. How many choices does the design team have for an arrangement of toppings on the first of these columns?

    There are 16 choose 4, or 1820 ways to select 4 toppings for the left column, and 24 ways to arrange each of these choices, for a total of 43680 possible topping arrangements in the left column. **2 points**