Counting Multisets Solutions

1. The "stars and bars" method is a technique to count the number of ways to distribute identical items into distinct bins. The stars represent the identical items, and the bars represent the dividers between the distinct bins. The number of ways to arrange k stars and n-1 bars is given by the combination formula $\binom{k+n-1}{k}$ or $\binom{k+n-1}{n-1}$.

2. This can be represented as choosing a multiset of size 4 (scoops) from a set of 10 distinct elements (flavors). Using stars and bars, we have k = 4 stars (scoops) and n = 10 bins (flavors). We need n-1 = 9 bars. The number of ways is $\binom{4+10-1}{4} = \binom{13}{4} = 715$ ways.

3. The general formula for the number of positive integer solutions to the equation is given by the combination: $\binom{n-1}{k-1}$. Here we have n = 17 (total sum) and k = 3 (variables a, b, c), so the number of positive integer solutions is: $\binom{17-1}{3-1} = \binom{16}{2} = 120$.

   For non-negative integers, the formula becomes $\binom{n+k-1}{k-1} = \binom{17+3-1}{3-1} = \binom{19}{2} = 171.$

   When a, b, c ≥ -5, we need to perform a substitution so we have non-negative variables. Let:

   a’ = a + 5
   b’ = b + 5
   c’ = c + 5

   We need to now solve the equation:

   a’ + b’ + c’ - 15 = 17
   a’ + b’ + c’ = 32

   So we have $\binom{32+3-1}{3-1} = \binom{34}{2} = 561.$