22/22 A+

Discrete Math Counting Test   Name Akshay Kuchibhatla

1. Let $S = \{1, 4, 9, 16, 25, 36, 49\}$. How many subsets does S have in total? How many subsets contain $\{4, 16, 36\}$? How many subsets of S of cardinality 4 contain at least one odd number?

a. S has $2^7 = 128$ subsets in total. This answer can be reached by using the multiplicative principle in conjunction with saying "yes" or "no" for every element of S.

b. Now, we only have to say yes or no to 4 elements, as 3 are already included. $2^4 = 16$ subsets.

c. Since there are only 3 even numbers in S, every subset with cardinality 4 is guaranteed to have at least one odd number. So, the answer is just $\binom{7}{4} = 35$ subsets

2. Students at ACC can participate in student clubs during their Archer's period. There are 5 clubs (including the Programming Club) that meet on Mondays and Fridays, and 9 clubs that meet on Tuesdays, Wednesdays, and Thursdays.

a. If students have to participate in all meetings of their chosen club to be members, how many different combinations of clubs does a student have to choose from?

First off, there are 5 clubs that meet on Mondays and Fridays. A student can choose any one of these 5, so that gives us 5 choices. The student has 9 choices for clubs that meet on Tuesdays, Wednesdays and Thursdays. Using the mult. principle, $9 \cdot 5 = 45$ combos

b. What if students only need to attend a club once a week to be a member. How many options do they have?

This would mean that for the 5 clubs, the students have a choice on whether to attend 2 clubs once a week each or one club twice a week. This can be represented by $\binom{5}{1} + \binom{5}{2} = 5 + 10 = 15$ choices. For the 9 clubs, they can attend either 1, 2 or 3 clubs which gives us $\binom{9}{1} + \binom{9}{2} + \binom{9}{3} = 9 + 36 + 84 = 129$ options. Lastly, we have to multiply these numbers like in part a to get $129 \cdot 15 = 1935$ options.

40320
6435
201600
209600
128000
19200 0 0
259459200

720
7
5040

5040
8
40320

560
137
697

1820
24
7280
36400
43680

3. In an attempt to clean up your room, you have purchased a new wall mounting bookshelf to put some of the 15 books by different authors that you have stacked in a corner. The new bookshelf is large enough to hold 8 books.

   a. How many ways can you choose 8 of your 15 books for your bookshelf and then arrange them alphabetically by author on your shelf?

There are $\binom{15}{8}$ ways to choose 8 books. However, since there are all different authors, there is only only 1 way to arran[ge] them alphabhetically. So, the final answer is $\binom{15}{8}$ = 6435 ways.

   b. If you arrange the 8 books randomly, instead of my author, how many arrangements are there for a given set of 8 books? How many possible arrangements altogether are there for 8 of 15 randomly selected books randomly arranged?

There are 8! = 40,320 ways to arrange ~~one~~ single set of 8 books. We can multiply this by the total subsets of 8 out of 15 to get 40,320 · 6,435 = 259,459,200 ways.

4. Bruni's Pizza in Hammonton, NJ, makes the best pizza in the world. They also offer 16 different choices for toppings.

   a. How many 2-topping pizzas could you order at Bruni's?

$\binom{16}{2}$ = 120 2-topping pizzas.

   a. How many different pizza experiences are possible altogether, if you can order plain cheese, or one, two, or three toppings?

That's $\binom{16}{0}+\binom{16}{1}+\binom{16}{2}+\binom{16}{3}$ = 1+16+120+560 = 697 pizzas.

   b. Bruni's wants NOVA Web Development to build them a new website listing their 16 toppings choices in 4 equally sized columns. How many choices does the design team have for an arrangement of toppings on the first of these columns?

4 equally-sized columns would mean 4 toppings per column. So, the first column can contain $\binom{16}{4}$ = 1820 combinations of toppings. However, the design team also cares about the arrangement of the 4 toppings in the first column, and there are 4! = 24 arrangements. We multiply these together to get 24 · 1820 = 43,680 total arrangem[ents]