Discrete Math Logic and Proofs Chapter Test

Name José Ejemplo

1. Complete the truth table for $(Q \vee \neg R) \rightarrow (P \wedge R)$.

P	Q	R	(Q or ~R) => (P and R)
True	True	True	True
True	True	False	False
True	False	True	True
True	False	False	False
False	True	True	False
False	True	False	False
False	False	True	True
False	False	False	False

2. Prove the statement "For all integers n, if n is odd, then $11n$ is odd" and identify the proof technique you are using.

   By the definition of odd number, $n = 2k + 1$ for some $k \in I$, where I is the set of integer, thus we have:

   $$11n = 22k + 11 = 2(11k + 5) + 1$$

   Since the set of integers is closed under multiplication and k is an integer, $11k + 5$ is an integer. Let's call it m. We now have:

   $$11n = 2m + 1$$

   which by definition is an odd number, and we have established that $11n$ is odd whenever n is odd.

3. Prove that if natural number x is a multiple of 5, then nx is a multiple of 5 for all natural numbers n.

   Since x is a multiple of 5, x can be written as $5k$ for some $k \in N$. Thus nx can be written as $n5k$. Using the commutative and associative properties for multiplication, this can be written as $5(nk)$, which by definition is a multiple of 5.

4. In the game Rock, Paper, Scissors, the operation **beats** is defined by the following table:

beats	Rock	Paper	Scissors
Rock	False	False	True
Paper	True	False	False
Scissors	False	True	False

Using the definition of a **transitive relation** given in our text, that a relation R on a set A is transitive provided for all $x, y, z \in A$, if xRy and yRz, then xRz, determine if beats is a transitive relation.

Since Rock beats Scissors, and Scissors beats Paper, but Rock does not beat Paper, the beats relation is not transitive.

5. Let $f: X \to Y$ be a function, and let A and B be subsets of Y. Prove that $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$.

Let x be an arbitrary element in the set $f^{-1}(A \cup B)$. By the definition of **inverse image**, $f(x) \in A \cup B$. By the definition of the union of two sets, $f(x) \in A$ **or** $f(x) \in B$. If $f(x) \in A$, $x \in f^{-1}(A)$ and if $f(x) \in B$, $x \in f^{-1}(B)$. Since x is in either $f^{-1}(A)$ or $f^{-1}(B)$, $x \in f^{-1}(A) \cup f^{-1}(B)$. Thus every element of $f^{-1}(A \cup B)$ is also an element of $f^{-1}(A) \cup f^{-1}(B)$, and $f^{-1}(A \cup B) \subseteq f^{-1}(A) \cup f^{-1}(B)$.

Now let x be an arbitrary element in the set $f^{-1}(A) \cup f^{-1}(B)$. $x \in f^{-1}(A)$ **or** $x \in f^{-1}(B)$. If $x \in f^{-1}(A)$, then $f(x) \in A$ by the definition of inverse image. Likewise if $x \in f^{-1}(B)$ then $f(x) \in B$. In either case, by the definition of the union of sets, $f(x) \in A \cup B$ and $x \in f^{-1}(A \cup B)$. Thus every element of $f^{-1}(A) \cup f^{-1}(B)$ is also an element of $f^{-1}(A \cup B)$, giving us $f^{-1}(A) \cup f^{-1}(B) \subseteq f^{-1}(A \cup B)$.

Since $f^{-1}(A \cup B) \subseteq f^{-1}(A) \cup f^{-1}(B)$ and $f^{-1}(A) \cup f^{-1}(B) \subseteq f^{-1}(A \cup B)$, $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$.