

Combinations and Permutations Quiz Solutions

1. $C(29, 7) = \binom{29}{7} = \frac{29!}{7!(29-7)!} = \frac{29!}{7! \cdot 22!} = \frac{29 \cdot 28 \cdot 27 \cdot 26 \cdot 25 \cdot 24 \cdot 23 \cdot 22}{7!} = \frac{7866331200}{5040} = 1560780$

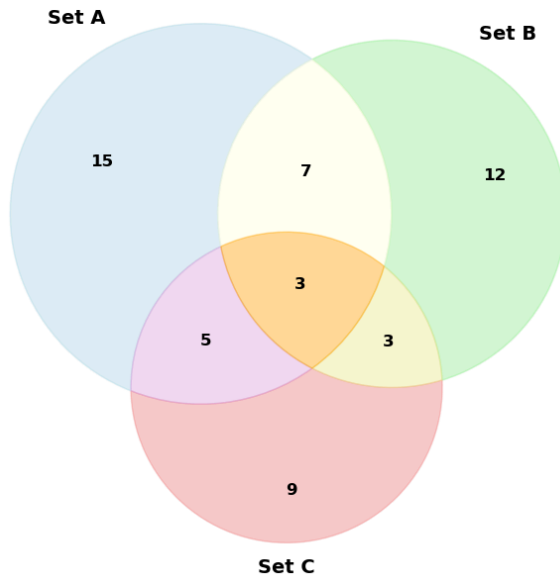
2. $16^6 = 16777216$ combinations. $P(16, 6) = \frac{16!}{(16-6)!} = 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 = 5765760$

without repeats (6-permutation of 16 elements). For decreasing order, there is only one way to arrange a string of unique digits in decreasing order, and we have

$$\binom{16}{6} = \frac{16!}{6!10!} = \frac{5765760}{6!} = 8008 \text{ possible strings.}$$

3. **Part(a)**. Here is a venn diagram showing A, B and C:

Venn Diagram: $|A|=30, |B|=25, |C|=20$
 $|A \cap B|=10, |A \cap C|=8, |B \cap C|=6$ and $|A \cap B \cap C|=3$



Here is how these values were calculated:

$$\begin{aligned} |A \cap B| - |A \cap B \cap C| &= 10 - 3 = 7 \\ |A \cap C| - |A \cap B \cap C| &= 8 - 3 = 5 \\ |B \cap C| - |A \cap B \cap C| &= 6 - 3 = 3 \\ |A| - [(A \cap B \text{ only}) + (A \cap C \text{ only}) + (A \cap B \cap C)] &= 30 - [7 + 5 + 3] = 30 - 15 = 15 \\ \text{B only: } 25 - [7 + 3 + 3] &= 25 - 13 = 12 \\ \text{C only: } 20 - [5 + 3 + 3] &= 20 - 11 = 9 \end{aligned}$$

Now, $(A \cup B) \setminus C$ consists of:

- A only: 15
- B only: 12
- $A \cap B$ only: 7

Sum: $15 + 12 + 7 = 34$

Method 2: Using PIE

We can also compute:

$$|(A \cup B) \setminus C| = |A \cup B| - |(A \cup B) \cap C|$$

Now,

$$|A \cup B| = |A| + |B| - |A \cap B| = 30 + 25 - 10 = 45$$

$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$

So

$$|(A \cup B) \cap C| = |A \cap C| + |B \cap C| - |A \cap B \cap C| = 8 + 6 - 3 = 11$$

Thus

$$|(A \cup B) \setminus C| = 45 - 11 = 34$$

Part(b)

We need to describe a set in terms of sets A, B, and C that has cardinality 14. We were given that set C has cardinality 20, and that the intersection of C and B has cardinality 6. Since:

$$20 - 6 = 14$$

We have:

$$|C \setminus B| = 14$$